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СИБИРСКИЕ ЭЛЕКТРОННЫЕ
МАТЕМАТИЧЕСКИЕ ИЗВЕСТИЯ
Siberian Electronic Mathematical Reports
http://semr.math.nsc.ru
ISSN 1813-3304

Vol. 23, No. 2, pp. 1078–1085 (2026)
https://doi.org/10.33048/semi.2026.23.065

УДК 512.54
MSC 20D06, 20E45

RECOGNIZING THE QUASISIMPLE GROUPS $\mathrm{SL}_2(q)$

ALIREZA KHALILI ASBOEI

Communicated by I.B. Gorshkov

Abstract: For a finite group G, let $\mathrm{cs}(G) = \{|x^G| : x \in G\}$ be the set of its conjugacy class sizes. We prove that if $q > 3$ is an odd prime power and H is a finite group such that $|H| = |\mathrm{SL}_2(q)|$ and $\mathrm{cs}(H) = \mathrm{cs}(\mathrm{SL}_2(q))$, then $H \cong \mathrm{SL}_2(q)$. Thus the pair consisting of the group order and the set of conjugacy class sizes recognizes the quasisimple group $\mathrm{SL}_2(q)$. We also show that the multiplicities of the conjugacy class sizes are forced by the same data.

Keywords: Conjugacy class size, finite group, quasisimple group, recognition problem, special linear group.

1 Introduction

Recognition of finite groups by arithmetic data has been studied from several points of view. One of the best known problems in this direction is Thompson's conjecture, which asks whether a finite centerless group having the same set of conjugacy class sizes as a nonabelian finite simple group must be isomorphic to that simple group. The conjecture has been proved for all finite simple groups except the alternating groups, while for alternating groups it is known modulo the binary Goldbach conjecture [13, 14]; see also [20]. Earlier contributions include [2, 10, 11]. For projective special linear groups, Chen and Chen obtained recognition results using the group order together with a special conjugacy class size [12]. Related recognition results based on

Alireza Khalili Asboei, Recognizing the Quasisimple Groups $\mathrm{SL}_2(q)$.
© 2026 Alireza Khalili Asboei.
Received September 4, 2026, Published November 23, 2026.

the group order and one conjugacy class size were obtained for alternating, symplectic, and projective special linear groups [4–8].

A stronger result for $\mathrm{PSL}_2(q)$ was proved by Akhlaghi et al.: for $q > 3$, equality of the sets of conjugacy class sizes already implies a direct product of $\mathrm{PSL}_2(q)$ with an abelian group [3]. Gorshkov subsequently proved the analogous statement for the smallest quasisimple case relevant here: if $\mathrm{cs}(G) = \mathrm{cs}(\mathrm{SL}_2(5))$, then $G \cong \mathrm{SL}_2(5) \times A$ for some abelian group A [15]. In particular, adding the condition $|G| = |\mathrm{SL}_2(5)|$ forces $A = 1$, so $\mathrm{SL}_2(5)$ is characterized by its order and the set of its conjugacy class sizes. The connection between the structure of a finite group and the set of its conjugacy class sizes has remained an active research topic; for a recent short account of results and open problems in this direction, see Vasil'ev and Gorshkov [19], and see also [14, 16].

Very recently, Gorshkov and Vasil'ev proved that every alternating or symmetric group is characterized among finite groups by its order and the set of its conjugacy class sizes [13, Theorem 1.1]. Together with the previously known results for finite simple groups, this yields the same recognition property for every finite simple group [13, Corollary 1.3]. Thus it is natural to ask how far the pair $(|G|, \mathrm{cs}(G))$ continues to determine groups beyond the simple case.

The quasisimple setting is subtler because a nontrivial center is present. The conjugacy classes of $\mathrm{SL}_2(q)$ themselves have been studied directly; see, for example, Adan-Bante and Harris [1] and the explicit tables in [9]. The available recognition results closest to our setting concern mainly the simple quotient $\mathrm{PSL}_2(q)$. In particular, [3] uses $\mathrm{SL}_2(q)$ as a possible central cover in the proof of the recognition theorem for $\mathrm{PSL}_2(q)$, but does not formulate the recognition statement proved here for $\mathrm{SL}_2(q)$ itself.

Vasil'ev recently formulated a systematic framework for arithmetic invariants of finite simple and related groups and asked, in particular, whether the pair consisting of the order and the set of conjugacy class sizes is a full system of invariants for quasisimple groups [21]. We give an affirmative answer for the infinite quasisimple family $\mathrm{SL}_2(q)$ when $q > 3$ is odd.

Theorem 1. *Let $q = p^f > 3$ be an odd prime power and let H be a finite group. If $|H| = |\mathrm{SL}_2(q)|$ and $\mathrm{cs}(H) = \mathrm{cs}(\mathrm{SL}_2(q))$, then $H \cong \mathrm{SL}_2(q)$.*

The proof first recovers the center from the assumed data. After passing to $K = H/Z(H)$, we show that its Sylow p-subgroups are elementary abelian of order q, pairwise intersect trivially, and form a set of size $q + 1$. Conjugation gives a faithful doubly transitive action on this set. The rank-one theorem of Hering et al. [18] identifies K with $\mathrm{PSL}_2(q)$, and a final central-extension argument recovers $\mathrm{SL}_2(q)$.

2 Preliminaries

All groups are finite. For $x \in G$, write x^G for its conjugacy class and $C_G(x)$ for its centralizer. We use $\mathrm{cs}(G) = \{|x^G| : x \in G\}$, and write $\mathrm{cs}^*(G)$ when multiplicities are retained. For a positive integer n, its p-part is denoted by n_p.

Throughout, $q = p^f > 3$ is an odd prime power and $L = \mathrm{SL}_2(q)$. The standard conjugacy class description gives $|L| = q(q^2-1)$ and $\mathrm{cs}(L) = \{1, (q^2-1)/2, q(q-1), q(q+1)\}$; see [9, Table 1]. Put $A = (q^2-1)/2$, $B = q(q-1)$ and $C = q(q+1)$.

Following Gorshkov [17], a finite group X belongs to $R(p)$ if there is an integer $\alpha > 0$ such that $|x^X|_p \in \{1, p^\alpha\}$ for every $x \in X$. We shall use the following consequence of [17, Corollary 1.5].

Lemma 1. *If $X \in R(p)$ and $Z(X) = 1$, then every Sylow p-subgroup of X is abelian.*

We shall also use [18, Theorem 1.1]: if a finite group acts doubly transitively on a set and a point stabilizer contains a normal subgroup acting regularly on the remaining points, then it has a normal subgroup whose induced action is one of the standard rank-one actions: sharply doubly transitive, $\mathrm{PSL}_2(r)$, $\mathrm{Sz}(r)$, $\mathrm{PSU}_3(r)$, or ${}^2G_2(r)$.

3 Main results

Assume that H satisfies the hypotheses of Theorem 1.

Lemma 2. *We have $|Z(H)| = 2$.*

Proof. The centralizer orders corresponding to the class sizes A, B, C are $2q, q+1, q-1$. Hence $|Z(H)|$ divides $\gcd(2q, q-1, q+1) = 2$. If $Z(H) = 1$, then for positive integers a, b, c the class equation is

$$q(q^2-1) = 1 + aA + bB + cC. \tag{1}$$

Since q is odd, A, B, C are even. Thus the right side of (1) is odd, whereas the left side is even, a contradiction. □

Write $Z(H) = \langle z \rangle$.

Lemma 3. *The numbers of classes of sizes A, B, C are $4, (q-1)/2, (q-3)/2$, respectively.*

Proof. Let these numbers be a, b, c. Then

$$q(q^2-1) = 2 + aA + bB + cC. \tag{2}$$

Reducing (2) modulo $q+1$ gives $b \equiv -1 \pmod{(q+1)/2}$. Since $bB < |H|$, we have $b < q+1$, and hence $b = (q-1)/2 + \varepsilon(q+1)/2$ with $\varepsilon \in \{0, 1\}$.

Similarly, reduction modulo $q-1$ gives $c \equiv -1 \pmod{(q-1)/2}$. Since $cC < |H|$, we have $c < q-1$, and hence $c = (q-3)/2 + \delta(q-1)/2$ with $\delta \in \{0, 1\}$. Substituting these expressions in (2) gives $a = 4 - q(\varepsilon + \delta)$. If

$\varepsilon+\delta \geq 1$, then $a \leq 4-q<0$, contrary to $a>0$. Hence $\varepsilon=\delta=0$. Therefore $b=(q-1)/2$, $c=(q-3)/2$, and $a=4$. □

Corollary 1. *The data* $|H|$ *and* $\mathrm{cs}(H)$ *determine* $\mathrm{cs}^*(H)$.

Lemma 4. *If* $x \in H$ *and* $|x^H|=A$, *then* $C_H(x)=Z(H)\times P$ *for a Sylow* p*-subgroup* P *of* $C_H(x)$, *and* $x^H \neq (xz)^H$.

Proof. We have $|C_H(x)|=2q$. Its number of Sylow p-subgroups divides 2 and is congruent to 1 modulo p, so it is 1. Thus its Sylow p-subgroup P is normal. Since $Z(H)\leq C_H(x)$, $P\cap Z(H)=1$, and $|Z(H)P|=2q$, we obtain $C_H(x)=Z(H)\times P$.

Since $C_H(x)=Z(H)\times P$, we have $Z(C_H(x))=Z(H)\times Z(P)$. Every element belongs to the center of its own centralizer, so $x\in Z(C_H(x))$. Hence $x=z^\varepsilon u$ with $u\in Z(P)$. As $x\notin Z(H)$, $u\neq 1$. The elements u and zu have orders p^e and $2p^e$ for some $e>0$, so they cannot be conjugate. Hence $x^H\neq (xz)^H$. □

Lemma 5. *Every nonidentity* p*-element of* H *lies in a class of size* A, *and exactly two of the four classes of size* A *consist of* p*-elements.*

Proof. If x is a nonidentity p-element, then p divides $|C_H(x)|$. Hence its class cannot have size B or C, whose centralizer orders are $q+1$ and $q-1$. Thus $|x^H|=A$. By Lemma 4, multiplication by z pairs the four A-classes without fixed points. In each pair we may choose representatives u and zu with $1\neq u\in Z(P)$ for a suitable Sylow p-subgroup P of the corresponding centralizer. The element u has p-power order, whereas zu has twice that order. Hence exactly one class in each pair consists of p-elements. □

Put $K=H/Z(H)$.

Lemma 6. *For* $x\in H$, *the class size of* $xZ(H)$ *in* K *is* $|x^H|$ *or* $|x^H|/2$. *It is* $|x^H|/2$ *if and only if* $x^H=(xz)^H$.

Proof. Let D be the preimage of $C_K(xZ(H))$ in H. Then D consists of the elements $g\in H$ such that $x^g\in\{x,xz\}$. Thus $C_H(x)\leq D$, and $|D:C_H(x)|=1$ if $x^H\neq(xz)^H$, while $|D:C_H(x)|=2$ if $x^H=(xz)^H$. Since $|K|=|H|/2$ and $|C_K(xZ(H))|=|D|/2$, we have

$$|(xZ(H))^K|=\frac{|H|}{|D|}. \tag{3}$$

Therefore the class size is $|x^H|$ in the first case and $|x^H|/2$ in the second. □

Lemma 7. *We have* $Z(K)=1$. *Moreover, every nontrivial conjugacy class of* K *has size at least* $(q^2-1)/4$.

Proof. Let $xZ(H)\in Z(K)$. Then $|(xZ(H))^K|=1$. By Lemma 6, this number is either $|x^H|$ or $|x^H|/2$. Hence $|x^H|\in\{1,2\}$. Since $2\notin\mathrm{cs}(H)$, we have $|x^H|=1$, so $x\in Z(H)$ and $xZ(H)=1$. Thus $Z(K)=1$. If $1\neq y\in K$, Lemma 6 shows that $|y^K|$ is one of $A, A/2, B, B/2, C, C/2$. The least of these values is $A/2=(q^2-1)/4$. □

Lemma 8. *The group K has exactly two nonidentity classes of p-elements. Each has size A, and every nonidentity p-element $x \in K$ satisfies $|C_K(x)| = q$. Thus K has $q^2 - 1$ nonidentity p-elements.*

Proof. Every p-element of K has a p-element lift to H. Indeed, if $hZ(H)$ has order p^e, then either $h^{p^e} = 1$ or $h^{p^e} = z$; in the latter case $(hz)^{p^e} = 1$ because p^e is odd. Lemma 5 therefore accounts for all nonidentity p-elements of K.

The four A-classes form two pairs under multiplication by z, and the two classes in each pair are distinct by Lemma 4. The two pairs have distinct images in K. Indeed, if u_1 and u_2 are p-element representatives from the two pairs and their images were conjugate in K, then some $g \in H$ would satisfy $u_1^g = u_2$ or $u_1^g = u_2 z$. The second equality is impossible because the two sides have odd and even orders, respectively, while the first would make u_1 and u_2 conjugate in H, contrary to the choice of distinct pairs.

In each pair the two H-classes are distinct, so Lemma 6 shows that their common image in K has class size A, not $A/2$. Since $|K| = q(q^2 - 1)/2$, its centralizer has order q. The total number of nonidentity p-elements is $2A = q^2 - 1$. □

Lemma 9. *Every Sylow p-subgroup of K is abelian.*

Proof. By Lemma 6, every conjugacy class size in K is either a class size of H or half of one. Since p is odd, division by 2 does not change the p-part. The p-parts of A, B, C are $1, q, q$, respectively. Hence, for every $x \in K$, we have $|x^K|_p \in \{1, q\}$. Thus $K \in R(p)$. By Lemma 7, $Z(K) = 1$, so Lemma 1 applies. □

Lemma 10. *Let $P \in \mathrm{Syl}_p(K)$. For every $x \in P \setminus \{1\}$, $C_K(x) = P$. Distinct Sylow p-subgroups intersect trivially, and K has exactly $q + 1$ Sylow p-subgroups.*

Proof. Since P is abelian, every element of P commutes with x, so $P \leq C_K(x)$. Lemma 8 gives $|C_K(x)| = q = |P|$, and therefore $C_K(x) = P$. If two Sylow p-subgroups P and Q contain the same nonidentity element x, then both are contained in $C_K(x) = P$, and hence $P = Q$. Thus distinct Sylow p-subgroups intersect trivially. By Lemma 8, there are $q^2 - 1$ nonidentity p-elements in K. Each Sylow p-subgroup contains $q - 1$ of them, and no nonidentity element is counted twice. Hence the number of Sylow p-subgroups is $(q^2 - 1)/(q - 1) = q + 1$. □

Lemma 11. *Let $P \in \mathrm{Syl}_p(K)$. Then $P \cong C_p^f$.*

Proof. By Lemma 10, K has $q + 1$ Sylow p-subgroups. Hence $|N_K(P)| = |K|/(q + 1) = q(q - 1)/2$, and therefore $|N_K(P) : P| = (q - 1)/2$. Every nonidentity p-class of K meets P, because a representative lies in some Sylow p-subgroup and all Sylow p-subgroups are conjugate. If $x, y \in P \setminus \{1\}$ and $x^g = y$, then $P^g = C_K(x)^g = C_K(y) = P$, so $g \in N_K(P)$. Thus the intersection of each K-class of nonidentity p-elements with P is exactly one $N_K(P)$-orbit. By Lemma 8, there are exactly two such orbits.

The action of $N_K(P)/P$ on $P \setminus \{1\}$ is semiregular: if $g \in N_K(P)$ fixes $x \neq 1$, then $g \in C_K(x) = P$. Hence each orbit has size $(q-1)/2$.

Since P is abelian, $\Omega_1(P) = \{x \in P : x^p = 1\}$ is characteristic in P. Because $P \triangleleft N_K(P)$, the subgroup $\Omega_1(P)$ is invariant under $N_K(P)$. It is nontrivial, so $\Omega_1(P) \setminus \{1\}$ is a nonempty union of the two orbits above. Consequently it is either one orbit or both. If $|\Omega_1(P)| = p^d$, then $p^d - 1 = (q-1)/2$ or $p^d - 1 = q - 1$. The first equality gives $2p^d = p^f + 1$. The left side is divisible by p, whereas the right side is congruent to 1 modulo p, a contradiction. Therefore $p^d = q$, so $\Omega_1(P) = P$. Thus P is abelian of order p^f and exponent p, and hence $P \cong C_p^f$. □

Lemma 12. *Let $P \in \mathrm{Syl}_p(K)$. The conjugation action of K on $\mathrm{Syl}_p(K)$ is faithful and doubly transitive, and P acts regularly on $\mathrm{Syl}_p(K) \setminus \{P\}$.*

Proof. If $1 \neq x \in P$ fixes another Sylow p-subgroup Q, then $x \in N_K(Q)$. Because $Q \triangleleft N_K(Q)$, it is the unique Sylow p-subgroup of $N_K(Q)$. Every p-element of $N_K(Q)$ therefore lies in Q, so $x \in Q$. Thus $x \in P \cap Q$, and Lemma 10 gives $P = Q$. Hence P acts semiregularly on the remaining q points. Every orbit of a semiregular action of P has size $|P| = q$, and there are exactly q remaining points. Thus the action is regular. In particular, the point stabilizer $N_K(P)$ is transitive on the remaining points, so the conjugation action of K is doubly transitive.

Let D be the kernel of this action. Since D fixes every Sylow p-subgroup, we have $D \leq N_K(P)$. Moreover, no nonidentity element of P fixes any point other than P, whereas every element of D fixes every point. Hence $D \cap P = 1$. Since $D \triangleleft K$, we have $[D, P] \leq D \cap P = 1$, so $D \leq C_K(P)$. For $1 \neq x \in P$, $C_K(P) \leq C_K(x) = P$. Thus $D \leq P$, and $D = 1$. □

Lemma 13. *We have $K \cong \mathrm{PSL}_2(q)$.*

Proof. By Lemma 12, K acts faithfully and doubly transitively on $\mathrm{Syl}_p(K)$, which has $q+1$ points. The point stabilizer $N_K(P)$ contains the normal subgroup P, and P acts regularly on the remaining q points. Thus all hypotheses of [18, Theorem 1.1] are satisfied. Let $M \triangleleft K$ be the normal subgroup supplied by that theorem.

If M is sharply doubly transitive, its Frobenius kernel R is characteristic in M, hence normal in K, and $|R| = q+1$. For $1 \neq x \in R$, we have $|x^K| \leq q$. But Lemma 7 gives $|x^K| \geq (q^2-1)/4$. Since $q > 3$ is odd, $q \geq 5$, and $(q^2-1)/4 > q$. This is a contradiction.

If $M \cong \mathrm{Sz}(r)$, its natural degree is $r^2 + 1$ with r a power of 2, so $q = r^2$, contrary to the oddness of q.

If $M \cong \mathrm{PSU}_3(r)$, its natural degree is $r^3 + 1$, so $q = r^3$. Put $d = (3, r+1)$. Since $|M| = r^3(r^3+1)(r^2-1)/d$, divisibility of $|M|$ into $|K|$ would require

$$[K : M] = \frac{d(r^2 + r + 1)}{2(r+1)} \tag{4}$$

to be an integer. Since $\gcd(r+1, r^2+r+1) = 1$, $d \leq 3$ and $r+1 \geq 4$, this is impossible.

If $M \cong {}^2G_2(r)$, its natural degree is r^3+1, so again $q = r^3$. Since $|M| = r^3(r^3+1)(r-1)$, we obtain

$$[K : M] = \frac{r^2+r+1}{2}. \tag{5}$$

The numerator is odd, so (5) is not an integer.

Thus $M \cong \mathrm{PSL}_2(r)$. Its natural degree is $r+1 = q+1$, so $r = q$. Hence $|M| = q(q^2-1)/2 = |K|$, and $K = M \cong \mathrm{PSL}_2(q)$. □

Lemma 14. *The group H is perfect.*

Proof. Since $H/Z(H) \cong \mathrm{PSL}_2(q)$ is perfect, $H = H'Z(H)$. If $H' \cap Z(H) = 1$, then $H \cong \mathrm{PSL}_2(q) \times C_2$. Let t be an involution of $\mathrm{PSL}_2(q)$. By [3, Remark 2.11], its class size is $q(q+1)/2$ or $q(q-1)/2$. Direct product with C_2 does not change class sizes, but neither value belongs to $\{1, A, B, C\}$. This is a contradiction. Hence $H' \cap Z(H) \neq 1$. Since $|Z(H)| = 2$, we obtain $Z(H) \leq H'$ and therefore $H = H'$. □

Proof of Theorem 1. By Lemmas 2, 13 and 14, H is a perfect central extension of $\mathrm{PSL}_2(q)$ by a group of order 2. For $q \neq 9$, the universal covering group of $\mathrm{PSL}_2(q)$ is $\mathrm{SL}_2(q)$; see [18, Lemma 3.1(vii)] and the corresponding use of the Schur cover in [3, Theorem 2.19]. Hence the universal property gives an epimorphism $\mathrm{SL}_2(q) \to H$. The two groups have the same order, so this epimorphism is an isomorphism.

For $q = 9$, $\mathrm{PSL}_2(9) \cong A_6$ has Schur multiplier of order 6 [18, Lemma 3.1(vii)]. The universal central extension maps onto H, and the kernel of this map has order 3. Its center, being the Schur multiplier, is an abelian group of order 6 and hence cyclic; therefore its subgroup of order 3 is unique. Thus there is a unique perfect central double cover of A_6. Since $\mathrm{SL}_2(9)$ is such a cover of $\mathrm{PSL}_2(9) \cong A_6$, we obtain $H \cong \mathrm{SL}_2(9)$. □

Corollary 2. *For every odd prime power $q > 3$, the pair $(|G|, \mathrm{cs}(G))$ is a full system of invariants at $G = \mathrm{SL}_2(q)$.*

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Alireza Khalili Asboei
Department of Mathematics Education, Farhangian University,
P.O. Box 14665-889, Tehran, Iran
Email address: khaliliasbo@yahoo.com