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ON THE DETERMINATION OF TWO COEFFICIENTS IN A MULTIDIMENTIONAL QUASILINEAR PARABOLIC EQUATION

K.I. ISCHENKO AND **S.V. POLYNTSEVA**

Dedicated to 85th birthday of academician Vladimir G. Romanov

Abstract: This article discusses the problem of determining two unknown coefficients in one multidimensional quasilinear parabolic equation. By means of overdetermination conditions, the initial inverse problem is reduced to the direct auxiliary Cauchy problem. Furthermore, using the weak approximation method, on the basis of sufficiently smooth input data, the solvability of the direct problem is established. The solution of the inverse problem is written out explicitly through the solution of a direct problem, on this basis, the existence and uniqueness theorem of the solution of the original inverse problem in the class of smooth bounded functions is proved. An example of input data satisfying the conditions of the theorem is given.

Keywords: inverse problem, coefficient inverse problem, Cauchy problem, quasilinear parabolic equation, overdetermination conditions, weak approximation method.

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1 Introduction

The class of inverse problems, that is difficult to study, is coefficient inverse problems, in which, along with the primary source, it is necessary to determine some physical properties of the process (coefficients).

Coefficient inverse problems for quasilinear parabolic equations describe, model and control nonlinear diffusion and filtration processes, are used to identify (thermophysical) characteristics (with a large change in the tempereture interval, the thermophysical characteristics of the medium depend on the temperature distribution) of the processes in thermophysics and mechanics of continuous media, etc.

Note that the definition of quasilinear parabolic equations is given in [1].

The problems of determining the coefficients of quasilinear parabolic equations were investigated by N.L. Gol'dman, K.T. Iskakov, A.D. Iskenderov, S.I. Kabanikhin, I.V. Koptyug, A. Lorenzi, R.Z. Sagdeev, V.M. Volkov [2] - [9] and others.

In [9], an inverse problem is considered for a quasilinear equation in a semi-infinite strip in the case of one unknown coefficient for the second derivative with respect to the variable x, depending on the solution of the equation.

In [2] - [6], [8] coefficient inverse problems for quasilinear parabolic equations in bounded domains are investigated.

The problem considered in [7] clearly demonstrates the practical application of the solution of the inverse problem for a quasilinear parabolic equation. Namely, mass transport of liquids which obeys the equation with liquid concentration - dependent diffusivity is considered and water concentration profiles measured experimeally in the course of drying of water-saturated porous alumina pellets are shown to be successfully modelled assuming exponential concentration dependence of diffusivity.

In the present paper, the problem of determining two unknown coefficients depending on time and the independent variable x in a multidimensional quasilinear parabolic equation is investigated for the unique solvability.

The study is conducted using the following method:

- on the basis of the overdetermination conditions, the initial inverse problem is reduced to an auxiliary direct Cauchy problem for a nonlinear loaded equation;

- solvability of the direct problem is proved using sufficiently smooth input data and the weak approximation method [10];

- the solution of the inverse problem is written out explicitly through the solution of the direct problem, on this basis the theorem of existence and uniqueness of the classical solution of the inverse problem in the class of smooth bounded functions for $t \in [0, t^*]$ is proved.

Previously, this method was used in the case of coefficient inverse problems for linear and semilinear parabolic equations, see, for example, [11], [12].

Also in the present paper, the condition for the dependence of t^* on the constants limiting the input data was written out, in [11], [12] this condition was not written out. An example of input data satisfying the conditions of the proved theorem of existence and uniqueness of the classical solution of the inverse problem is given.

2 Problem statement

In $G_{[0,T]} = \{(t,x,z)\big|0 \leqslant t \leqslant T, x \in E_n, z \in E_1\}$ we have the Cauchy problem

$$\frac{\partial u}{\partial t} = L_x(u) + a(t,x)uu_{zz} + \beta_1(t,x)u_z + \beta_2(t,x)u^2 + b(t,x)f(t,x,z), \quad (1)$$

$$u(0,x,z) = u_0(x,z), \quad (x,z) \in E_{n+1}. \quad (2)$$

Here

$$L_x(u) = \sum_{i,j=1}^{n} \alpha_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} \alpha_i \frac{\partial u}{\partial x_i},$$

functions $f(t,x,z)$ and $u_0(x,z)$ are given in $G_{[0,T]}$ and E_{n+1} respectively, coefficients α_{ij}, α_i, $i,j = \overline{1,n}$, $\beta_1(t,x)$, $\beta_2(t,x)$ are continuously differentiable, real-valued functions of t, and t,x, respectively, $0 \leqslant t \leqslant T > 0$, $T - const$, E_n is the n-dimensional Euclidian space, $n \geqslant 1$, $n \in \mathbf{N}$.

Let be $\alpha_{ij}(t) = \alpha_{ji}(t)$ and the ratio

$$\sum_{i,j=1}^{n} \alpha_{ij}\xi_i\xi_j > 0 \quad \forall \xi \in E_n \backslash \{0\}, \quad t \in [0,T],$$

is satisfied.

We assume that overdetermination conditions on two different hypersurface $z = d_1(t)$ and $z = d_2(t)$ are hold:

$$u(t,x,d_1(t)) = \phi_1(t,x), \quad u(t,x,d_2(t)) = \phi_2(t,x), \quad (t,x) \in \Pi_{[0,T]}, \quad (3)$$

where $\Pi_{[0,T]} = \{(t,x)|\, 0 \leqslant t \leqslant T, x \in E_n\}$, $d_1(t) \neq d_2(t)$ and $\phi_1(t,x)$, $\phi_2(t,x)$ — are given functions, which satisfy matching conditions

$$\phi_1(0,x) = u_0(x,d_1(0)), \quad \phi_2(0,x) = u_0(x,d_2(0)). \quad (4)$$

The solution to the inverse problem (1) – (3) is a triple of functions $u(t,x,z)$, $a(t,x)$, $b(t,x)$, that belong to the class $Z(t^*)$, $0 < t^* \leqslant T$, see (7), and satisfy relations (1) – (3).

3 Main results

In the presented paper a following the existence and uniqueness theorem of the solution of the original inverse problem is proved:

Theorem. *Let conditions* (4),

$$
\begin{aligned}
N_1(0,x)u_0(x,z) &= (P(0,x)f(0,x,d_2(0))-\\
&\quad - Q(0,x)f(0,x,d_1(0)))u_0(x,z) \geqslant \delta_1,\\
N_2(0,x) &= \phi_1(0,x)\frac{\partial^2 u_0(x,d_1(0))}{\partial z^2}f(0,x,d_2(0))-\\
&\quad - \phi_2(0,x)\frac{\partial^2 u_0(x,d_2(0))}{\partial z^2}f(0,x,d_1(0)) \geqslant \delta_2,
\end{aligned}
\tag{5}
$$

where $\delta_1,\ \delta_2 > 0,\ \delta_1,\ \delta_2 =$ *const,*

$$
\begin{aligned}
P(0,x) &= \phi_{1t}(0,x) - L_x(\phi_1(0,x)) - (\beta_1(0,x) + d_1'(0))u_{0z}|_{z=d_1(0)} -\\
&\quad -\beta_2(0,x)\phi_1^2(0,x),\\
Q(0,x) &= \phi_{2t}(0,x) - L_x(\phi_2(0,x)) - (\beta_1(0,x) + d_2'(0))u_{0z}|_{z=d_2(0)} -\\
&\quad -\beta_2(0,x)\phi_2^2(0,x),
\end{aligned}
$$

be fulfilled. It is assumed that the input data are smooth enough, have all continuous derivatives included in the following relation

$$
\begin{aligned}
&|D_x^\gamma \frac{\partial^{l_1}}{\partial z^{l_1}}\frac{\partial^g}{\partial t^g}f(t,x,z)| + |D_x^\gamma \frac{\partial^g}{\partial t^g}L_x(\phi_{g_1}(t,x))| +\\
&\quad + |D_x^\gamma \frac{\partial^{l_1}}{\partial z^{l_1}}u_0(x,z)| + |D_x^\gamma \frac{\partial^g}{\partial t^g}\beta_{g_1}(t,x)| +\\
&\quad + |D_x^\gamma \frac{\partial^g}{\partial t^g}\phi_{g_1}(t,x)| + |\frac{d^{s_1}}{dt^{s_1}}d_{g_1}(t)| \leqslant C,
\end{aligned}
\tag{6}
$$

here $(t,x,z) \in G_{[0,T]},\ l_1 = \overline{0, 10-2|\gamma|},\ |\gamma| \leqslant 4,\ g = 0,1,\ g_1 = 1,2,\ s_1 = 1,2,$ $\gamma = (\gamma_1, ..., \gamma_n)$ *-multi-index,* $D_x^\gamma = \frac{\partial^{|\gamma|}}{\partial x_1^{\gamma_1}...\partial x_n^{\gamma_n}}$, $|\gamma| = \sum\limits_{i=0}^{n}\gamma_i$, $C \geqslant 1$ *- constant. It should be noted that here and below* C *are different constants. Then there exists a unique solution* $u(t,x,z),\ a(t,x),\ b(t,x)$ *of the problem* (1) - (3) *in*

$$
\begin{aligned}
Z(t^*) = \{u(t,x,z), a(t,x), b(t,x) | u \in C^{1,2,4}_{t,x,z}(G_{[0,t^*]}),\\
a(t,x), b(t,x) \in C^{0,2}_{t,x}(\Pi_{[0,t^*]})\},
\end{aligned}
\tag{7}
$$

satisfies inequalities

$$
\sum_{|\beta|\leqslant 2}\sum_{k=0}^{4}\Big|D_x^\beta \frac{\partial^k}{\partial z^k}u(t,x,z)\Big| \leqslant C, \quad (t,x,z) \in G_{[0,t^*]},
\tag{8}
$$

$$
\sum_{|\beta|\leqslant 2}\Big|D_x^\beta a(t,x)\Big| + \sum_{|\beta|\leqslant 2}\Big|D_x^\beta b(t,x)\Big| \leqslant C, \quad (t,x) \in \Pi_{[0,t^*]},
\tag{9}
$$

where the class $C^{1,2,4}_{t,x,z}(G_{[0,t^*]})$ *is defined as follows*

$$C^{1,2,4}_{t,x,z}(G_{[0,t_*]}) = \{f_1(t,x,z)|\frac{\partial^g}{\partial t^g}f_1 \in C(G_{[0,t_*]})),$$
$$D^\beta_x \frac{\partial^k}{\partial z^k} f_1 \in C(G_{[0,t_*]}), |\beta| \le 2, k = \overline{0,4}, g = 0,1\}, \tag{10}$$

and

$$C^{0,2}_{t,x}(\Pi_{[0,t^*]}) = \{a_1(t,x)|D^\beta_x a_1(t,x) \in C(\Pi_{[0,t^*]}), |\beta| \leqslant 2\}.$$

The constant is $t^* = \min\Big(t_*, \frac{\delta_1}{2K_1}, \frac{\delta_2}{2K_2}\Big)$, *where constants* K_1, K_2 *depends on* C, δ_1, δ_2, *from relations* (5), (6), t_* *satisfies the inequality*

$$e^{60Ct_*(U(0)+1)^{20}e^{Ct_*(1+U(0))^{20}}} \leqslant 2,$$

here $U(0)$ *from (21) with* $t = 0$, *the constant* C *depend on* C, δ_2, *from* (5), (6).

The proof of this theorem is given in section 4.

An example of input data satisfying the conditions of theorem is given in section 5.

4 Proof of the theorem

By means of overdetermination conditions (3) and equation (1) we receive the system of linear algebraic equations by solving which we get form of the coefficients

$$a(t,x) = \frac{P(t,x)f(t,x,d_2(t)) - Q(t,x)f(t,x,d_1(t))}{\phi_1(t,x)u_{zz}|_{z=d_1(t)}f|_{z=d_2(t)} - \phi_2(t,x)u_{zz}|_{z=d_2(t)}f|_{z=d_1(t)}},$$
$$b(t,x) = \frac{Q(t,x)\phi_1(t,x)u_{zz}|_{z=d_1(t)} - P(t,x)\phi_2(t,x)u_{zz}|_{z=d_2(t)}}{\phi_1(t,x)u_{zz}|_{z=d_1(t)}f|_{z=d_2(t)} - \phi_2(t,x)u_{zz}|_{z=d_2(t)}f|_{z=d_1(t)}}. \tag{11}$$

Substituting (11) into (1), we arrive at the auxiliary direct problem:

$$u_t = L_x(u) + \frac{N_1}{N_2}uu_{zz} + \beta_1(t,x)u_z + \beta_2(t,x)u^2 + \frac{N_3}{N_2}f(t,x,z), \tag{12}$$

$$u(0,x,z) = u_0(x,z), \tag{13}$$

here

$$\begin{aligned}
&N_1 = N_1(t,x) = P(t,x)f(t,x,d_2(t)) - Q(t,x)f(t,x,d_1(t)),\\
&N_2 = N_2(t,x) = \phi_1(t,x)u_{zz}|_{z=d_1(t)}f|_{z=d_2(t)}-\\
&\quad - \phi_2(t,x)u_{zz}|_{z=d_2(t)}f|_{z=d_1(t)},\\
&N_3 = N_3(t,x) = Q(t,x)\phi_1(t,x)u_{zz}|_{z=d_1(t)}-\\
&\quad - P(t,x)\phi_2(t,x)u_{zz}|_{z=d_2(t)},\\
&P = P(t,x) = \phi_{1t}(t,x) - L_x(\phi_1(t,x)) - (\beta_1(t,x) + d_1'(t))u_z|_{z=d_1(t)}-\\
&\quad - \beta_2(t,x)\phi_1^2(t,x),\\
&Q = Q(t,x) = \phi_{2t}(t,x) - L_x(\phi_2(t,x)) - (\beta_1(t,x) + d_2'(t))u_z|_{z=d_2(t)}-\\
&\quad - \beta_2(t,x)\phi_2^2(t,x).
\end{aligned} \tag{14}$$

In order for the coefficient of the highest derivative to be positive and the denominators in the coefficients do not equal zero, we introduce the cutoff function $S_\delta(y) \in C^{10}(E_1)$, with the following properties

$$S_\delta(y) \geqslant \frac{\delta}{3} > 0,\ S_\delta(y) = \begin{cases} y,\ y \geqslant \frac{\delta}{2}, \\ \chi(y),\ \frac{\delta}{3} < y < \frac{\delta}{2}, \\ \frac{\delta}{3},\ y \leqslant \frac{\delta}{3}, \end{cases} \tag{15}$$

where $y \in E_1,\ \delta = const,\ \chi(y) \in C^{10}(E_1)$.

So we get following direct problem (16), (13)

$$\begin{aligned} u_t = L_x(u) + \frac{S_{\delta_1}(N_1(t,x)u)}{S_{\delta_2}(N_2(t,x))}u_{zz} + \beta_1(t,x)u_z + \\ +\beta_2(t,x)u^2 + \frac{N_3(t,x)}{S_{\delta_2}(N_2(t,x))}f(t,x,z). \end{aligned} \tag{16}$$

Let us prove the existence of a solution to the direct problem (16), (13). We apply the weak approximation method [10]. We split the problem (16), (13) and linearize it by a time shift by $\frac{\tau}{3}$ in the nonlinear terms

$$u_t^\tau = 3L_x(u^\tau), s\tau < t \leqslant (s+\frac{1}{3})\tau, \tag{17}$$

$$\begin{aligned} u_t^\tau = 3(\frac{S_{\delta_1}(N_1^\tau(t,x)u^\tau(t-\frac{\tau}{3}))}{S_{\delta_2}(N_2^\tau(t,x))}u_{zz}^\tau + \\ +\beta_1(t,x)u_z^\tau), (s+\frac{1}{3})\tau < t \leqslant (s+\frac{2}{3})\tau, \end{aligned} \tag{18}$$

$$\begin{aligned} u_t^\tau = 3(\beta_2(t,x)u^\tau u^\tau(t-\frac{\tau}{3}) + \\ +\frac{N_3^\tau(t,x)}{S_{\delta_2}(N_2^\tau(t,x))}f(t,x,z)), (s+\frac{2}{3})\tau < t \leqslant (s+1)\tau, \end{aligned} \tag{19}$$

$$u^\tau(0,x,z) = u_0(x,z), \quad x \in E_n, z \in E_1, \tag{20}$$

here $s = 0,1,...,N-1, \tau N = T, N > 0, N \in \mathbf{Z},\ u^\tau = u^\tau(t) = u^\tau(t,x,z)$,

$$N_1^\tau = N_1^\tau(t,x) = P^\tau(t,x)f(t,x,d_2(t)) - Q^\tau(t,x)f(t,x,d_1(t)),$$

$$N_2^\tau = N_2^\tau(t,x) = \phi_1(t,x)u_{zz}^\tau(t-\frac{\tau}{3},x,d_1(t))f|_{z=d_2(t)} -$$

$$-\phi_2(t,x)u_{zz}^\tau(t-\frac{\tau}{3},x,d_2(t))f|_{z=d_1(t)},$$

$$N_3^\tau = N_3^\tau(t,x) = Q^\tau(t,x)\phi_1(t,x)u_{zz}^\tau(t-\frac{\tau}{3},x,d_1(t)) -$$
$$-P^\tau(t,x)\phi_2(t,x)u_{zz}^\tau(t-\frac{\tau}{3},x,d_2(t)),$$
$$P^\tau = P^\tau(t,x) = \phi_{1t}(t,x) - L_x(\phi_1(t,x)) - (\beta_1(t,x)+d_1'(t))\cdot$$
$$\cdot u_z^\tau(t-\frac{\tau}{3},x,d_1(t)) - \beta_2(t,x)\phi_1^2(t,x),$$
$$Q^\tau = Q^\tau(t,x) = \phi_{2t}(t,x) - L_x(\phi_2(t,x)) - (\beta_1(t,x)+d_2'(t))\cdot$$
$$\cdot u_z^\tau(t-\frac{\tau}{3},x,d_2(t)) - \beta_2(t,x)\phi_2^2(t,x).$$

We introduce the notation

$$U^{\tau,t_0}(t) = \sum_{k=0}^{10} U_k^{\tau,t_0}(t), \tag{21}$$

$$U_k^{\tau,t_0}(t) = \sup_{t_0<\xi\leq t}\ \sup_{x\in E_n, z\in E_1}\left|\frac{\partial^k}{\partial z^k}u^\tau(\xi,x,z)\right|,$$
$$U_k(0) = \sup_{x\in E_n, z\in E_1}\left|\frac{\partial^k}{\partial z^k}u_0(x,z)\right|, \tag{22}$$

$$U_k^{\tau,t_0}(t_0) = \sup_{x\in E_n, z\in E_1}\left|\frac{\partial^k}{\partial z^k}u^\tau(t_0,x,z)\right|,\ t\in(t_0,(n+\frac{p}{3})\tau],$$
$$t_0\in[0,(n+\frac{p}{3})\tau),\ t>t_0,\ p=1,2,3. \tag{23}$$

The functions $U_k^{\tau,t_0}(t)$, $U_k^{\tau,t_0}(t_0)$, $U_k(0)$ are nonnegative and non-decreasing on each half-interval $(s\tau,(s+1)\tau]$.

Let us prove the a priori estimates guaranteeing the compactness of a set of solutions $\{u^\tau(t,x,z)\}$ of the problem (17) – (20).

Let the half-interval $(s\tau,(s+1)\tau]$ be s-th time step, where $s=\overline{0,N-1}$.

We consider the zero integer step $(s=0)$.

At the first fractional step $(p=1)$, we obtain the following estimate for the solution u^τ of problem (17), (20), due to (6) and the maximum principle [13]

$$|u^\tau(\xi,x,z)| \leqslant \sup_{x\in E_n, z\in E_1}|u_0(x,z)|, \quad 0<\xi\leqslant\frac{\tau}{3}. \tag{24}$$

We obtain the following estimates using differentiating equations (17), (20) with respect to z from one to ten times, respectively, due to (6) and the maximum principle [13]

$$\left|\frac{\partial^k}{\partial z^k}u^\tau(\xi,x,z)\right| \leqslant \sup_{x\in E_n, z\in E_1}\left|\frac{\partial^k}{\partial z^k}u_0(x,z)\right|, \quad k=\overline{1,10}, \quad 0<\xi\leqslant\frac{\tau}{3}. \tag{25}$$

We obtain the following estimate from (24), (25) through (21), (22)

$$U^{\tau,0}(t) \leqslant U(0), \qquad 0<t\leqslant\frac{\tau}{3}. \tag{26}$$

At the second fractional step ($p = 2$), we obtain the following estimate for the solution of equation (18) with initial data $u^\tau(\frac{\tau}{3}, x, z)$ due to (15), (6), (21) – (23) and the maximum principle [13]

$$U^{\tau,\frac{\tau}{3}}(t) \leqslant (1 + U^{\tau,\frac{\tau}{3}}(\frac{\tau}{3}))e^{C\tau(1+U^{\tau,\frac{\tau}{3}}(\frac{\tau}{3}))^{20}} - 1, \frac{\tau}{3} < t \leqslant \frac{2\tau}{3}. \qquad (27)$$

Collectively, due to (26), (27) on the first and second fractional steps, we get

$$U^{\tau,0}(t) \leqslant (1 + U(0))e^{C\tau(1+U(0))^{20}} - 1, 0 < t \leqslant \frac{2\tau}{3}. \qquad (28)$$

At the third fractional step ($p = 3$), integrating the equation (19) with $t \in \left(\frac{2\tau}{3}, \xi\right]$, $\frac{2\tau}{3} < \xi \leqslant \tau$, we receive the equality

$$u^\tau(\xi) = u^\tau(\frac{2\tau}{3}) + 3\int_{\frac{2\tau}{3}}^{\xi} (\beta_2(\eta, x)u^\tau(\eta)u^\tau(\eta - \frac{\tau}{3}, x, z) + \\ + \frac{N_3^\tau(\eta, x)}{S_{\delta_2}(N_2^\tau(\eta, x))} f(\eta, x, z))d\eta.$$

The last relation implies the inequality

$$|u^\tau(\xi)| \leqslant |u^\tau(\frac{2\tau}{3})| + 3\int_{\frac{2\tau}{3}}^{\xi} (|\beta_2(\eta, x)|u^\tau(\eta)||u^\tau(\eta - \frac{\tau}{3})| + \\ + \frac{|N_3^\tau(\eta, x)|}{|S_{\delta_2}(N_2^\tau(\eta, x))|}|f(\eta, x, z)|)d\eta,$$

where $\frac{2\tau}{3} < \xi \leqslant t \leqslant \tau$.

Since this inequality holds for all x, z we replace the functions of the integral terms by their exact upper bounds with respect to $x \in E_n, z \in E_1$, and then replace the function $|u^\tau|$, on the left-hand side of the inequality by $\sup\limits_{x\in E_n z\in E_1} |u^\tau|$, using notations (23), we obtain

$$U_0^{\tau,\frac{2\tau}{3}}(t) \leqslant U_0^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3}) + C\int_{\frac{2\tau}{3}}^{t} (U_0^{\tau,\frac{2\tau}{3}}(\eta)U_0^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3}) + \\ + U_2^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3}) + U_2^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3})U_1^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3}))d\eta. \qquad (29)$$

Further, in the same way, differentiating equations (19) with respect to z from one to 10 times, we get

$$U_k^{\tau,\frac{2\tau}{3}}(t) \leqslant U_k^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3}) + C\int_{\frac{2\tau}{3}}^{t} \sum_{q=0}^{k} (U_{k-q}^{\tau,\frac{2\tau}{3}}(\eta)U_q^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3}) + \\ + U_2^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3}) + U_2^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3})U_1^{\tau,\frac{2\tau}{3}}(\eta - \frac{\tau}{3}))d\eta, \quad k = \overline{1, 10}. \qquad (30)$$

Adding (29) and (30), by virtue of (21) we receive

$$U^{\tau,\frac{2\tau}{3}}(t) \leqslant U^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3}) + C\int_{\frac{2\tau}{3}}^{t}(U^{\tau,\frac{2\tau}{3}}(\eta)U^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3}) + U^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3})+ + U^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3})U^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3}))d\eta,$$

where $C \geqslant 1$-constant, independent of τ.

To the last inequality we apply the Gronwall lemma [14], then

$$U^{\tau,\frac{2\tau}{3}}(t) \leqslant (U^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3}) + 1)e^{2C\tau(U^{\tau,\frac{2\tau}{3}}(\frac{2\tau}{3})+1)} - 1, \quad \frac{2\tau}{3} < t \leqslant \tau.$$

Consequently, due to (28) and the last inequality at the zero whole step the following estimate holds

$$U^{\tau,0}(t) \leqslant (U(0)+1)e^{3C\tau(U(0)+1)^{20}e^{C\tau(1+U(0))^{20}}} - 1, \quad 0 < t \leqslant \tau.$$

Repeating similar arguments at the first whole step, we obtain

$$U^{\tau,\tau}(t) \leqslant (U^{\tau,\tau}(\tau)+1)e^{3C\tau(U^{\tau,\tau}(\tau)+1)^{20}e^{C\tau(1+U^{\tau,\tau}(\tau))^{20}}} - 1, \quad \tau < t \leqslant 2\tau.$$

Assuming that τ is sufficiently small and the inequality $e^{60C\tau(U(0)+1)^{20}e^{C\tau(1+U(0))^{20}}} \leqslant 2$ holds, at the zero and first whole steps we get

$$U^{\tau,0}(t) \leqslant (U(0)+1)e^{9C\tau(U(0)+1)^{20}e^{2C\tau(1+U(0))^{20}}} - 1, \quad 0 < t \leqslant 2\tau.$$

Analogous reasoning, at the $s-$th whole step ($s < N$) we obtain

$$U^{\tau,s\tau}(t) \leqslant (U^{\tau,s\tau}(s\tau)+1)e^{3C\tau(U^{\tau,s\tau}(s\tau)+1)^{20}e^{C\tau(U^{\tau,s\tau}(s\tau)+1)^{20}}} - 1,$$

$s\tau < t \leqslant (s+1)\tau$.

Consequently, at s whole steps, we getting

$$U^{\tau,0}(t) \leqslant (U(0)+1)e^{3(2s+1)C\tau(U(0)+1)^{20}e^{\psi(s)C\tau(1+U(0))^{20}}} - 1, \quad 0 < t \leqslant (s+1)\tau,$$

$$\psi(s) = \begin{cases} 1, \ s = 0, \\ 2, \ s = 1, 2, ..., N-1. \end{cases}$$

Hence, the following estimate is true

$$U^{\tau,0}(t) \leqslant (U(0)+1)e^{3Ct_*(U(0)+1)^{20}e^{t_*C(1+U(0))^{20}}} - 1, \quad 0 < t \leqslant t_*,$$

where t_* satisfies the inequality

$$e^{60Ct_*(U(0)+1)^{20}e^{Ct_*(1+U(0))^{20}}} \leqslant 2. \tag{31}$$

And, therefore, taking into account the notation (21), (22) uniformly with respect to τ

$$|\frac{\partial^k}{\partial z^k}u^\tau(t,x,z)| \leqslant C, \quad k = \overline{0,10}, \quad (t,x,z) \in G_{[0,t_*]}. \tag{32}$$

After differentiating problem (17) – (20) with respect to x_i, x_j, x_l and x_m, $i,j,l,m = \overline{1,n}$, we obtain equations that can be regarded as linear with coefficients uniformly bounded in τ. Arguing by analogy, considering (32), we obtain estimate

$$|D_x^\gamma \frac{\partial^k}{\partial z^k} u^\tau(t,x,z)| \leqslant C, \quad k=\overline{0,10-2|\gamma|},\ |\gamma| \leqslant 4, (t,x,z) \in G_{[0,t_*]}. \tag{33}$$

We obtain from (33) and (17) – (20) uniformly with respect to τ

$$|u_t^\tau(t,x,z)| \leqslant C, \quad (t,x,z) \in G_{[0,t_*]}.$$

We differentiate equations (17) – (20) once with respect to z. By (33), the right-hand side of the equations obtained is uniformly bounded in τ, and consequently the left-hand side is also uniformly bounded in τ

$$|u_{tz}^\tau(t,x,z)| \leqslant C, \quad (t,x,z) \in G_{[0,t_*]}.$$

By analogy, uniformly with respect to τ

$$|\frac{\partial^k}{\partial z^k} D_x^\lambda u_t^\tau(t,x,z)| \leqslant C, \quad k=\overline{0,4}, \quad |\lambda| \leqslant 2, \quad (t,x,z) \in G_{[0,t_*]}.$$

Thus, the following estimate holds uniformly with respect to τ

$$\begin{gathered} |\frac{\partial}{\partial t}\frac{\partial^k}{\partial z^k} D_x^\lambda u^\tau(t,x,z)| + |\frac{\partial}{\partial x_i}\frac{\partial^k}{\partial z^k} D_x^\lambda u^\tau(t,x,z)| + \\ + |\frac{\partial}{\partial z}\frac{\partial^k}{\partial z^k} D_x^\lambda u^\tau(t,x,z)| \leqslant C, k=\overline{0,4}, |\lambda| \leqslant 2, (t,x,z) \in G_{[0,t_*]}. \end{gathered} \tag{34}$$

The estimate (33) implies the uniform boundedness in τ of the family $\left\{D_x^\gamma \frac{\partial^k}{\partial z^k} u^\tau\right\}$ in $G_{[0,t_*]}$, and from (33), (34) their equicontinuity with respect to t, x and z is equicontinuous in $G_{[0,t_*]}$. Therefore, for any fixed γ, k, $|\gamma| \le 2$, $k=\overline{0,4}$, by the Arzela theorem [15] the set $\left\{D_x^\gamma \frac{\partial^k}{\partial z^k} u^\tau\right\}$ is compact in $C(G^M_{[0,t_*]})$, $M>0$ is an integer, $G^M_{[0,t_*]} = \{(t,x,z) | t \in [0,T], |x| \leqslant M, |z| \leqslant \ M\}$.

In a diagonal way, we choose a subsequence $\{u^\tau\}$ (we do not change the notation) converging together with the corresponding derivatives with respect to x and z to some function u in $G_{[0,t_*]}$, and also uniformly in each $G^M_{[0,t_*]}$. The function u is continuous, has derivatives of the corresponding order with respect to x and z that are continuous in $G_{[0,t_*]}$, and satisfies the initial data (2) and inequality

$$\left| D_x^\beta \frac{\partial^k}{\partial z^k} u(t,x,z) \right| \leqslant C,\ k=\overline{0,4},\ |\beta| \le 2,\ (t,x,z) \in G_{[0,t_*]}. \tag{35}$$

Since $D_\alpha^\gamma \frac{\partial^k}{\partial z^k} u^\tau \rightrightarrows D_x^\gamma \frac{\partial^k}{\partial z^k} u$ on $G^M_{[0,t_*]}$ $\forall M>0$, $|\gamma| \le 2$, $k=\overline{0,4}$ and the inequality (35), it can be proved (the proof is similar to the proof of Theorem 1.4 of the weak approximation method [10]) that the function u is a solution of the problem (16), (13) in $G^M_{[0,t_*]}$ for any fixed M, and since M is arbitrary, it is also in $G_{[0,t_*]}$.

The function $u(t,x,z)$ belongs to the class (10) and the estimate (35) is true.

In order that to prove the existence of a solution of problem (12), (13), it is necessary to remove the cutoff functions in equation (16). For this, we prove that for $(t,x,z) \in G_{[0,t^*]}$,

$$N_1(t,x)u(t,x,z) \geqslant \frac{\delta_1}{2}, \quad N_2(t,x) \geqslant \frac{\delta_2}{2}.$$

We differentiate $N_1(t,x)u(t,x,z)$ and $N_2(t,x)$ ($N_1(t,x)$, $N_2(t,x)$ from (14)) with respect to t

$$\begin{aligned}
M_1(t,x) &= (N_1(t,x)u(t,x,z))'_t = u(t,x,z)\big(P'_t f(t,x,d_2(t)) + \\
&+ P\big(f'_t(t,x,d_2(t)) + f'_z(t,x,d_2(t))d'_2(t)\big) - Q'_t f(t,x,d_1(t)) - \\
&- Q\big(f'_t(t,x,d_1(t)) + f'_z(t,x,d_1(t))d'_1(t)\big)\big) + \\
&+ \big(P(t,x)f(t,x,d_2(t)) - Q(t,x)f(t,x,d_1(t))\big)u_t(t,x,z), \\
M_2(t,x) &= (N_2(t,x))'_t = \big(u_{zzt}(t,x,d_1(t)) + u_{zzz}(t,x,d_1(t))d'_1(t)\big)\cdot \\
&\cdot f(t,x,d_2(t))\phi_1(t,x) + u_{zz}(t,x,d_1(t))\big(f'_t(t,x,d_2(t)) + \\
&+ f'_z(t,x,d_2(t))d'_2(t)\big)\phi_1(t,x) + u_{zz}(t,x,d_1(t))f(t,x,d_2(t))\cdot \\
&\cdot \phi_{1t}(t,x) - \big(f'_t(t,x,d_1(t)) + f'_z(t,x,d_1(t))d'_1(t)\big)\cdot \\
&\cdot u_{zz}(t,x,d_2(t))\phi_2(t,x) - f(t,x,d_1(t))\big(u_{zzt}(t,x,d_2(t)) + \\
&+ u_{zzz}(t,x,d_2(t))d'_2(t)\big)\phi_2(t,x) - \\
&- u_{zz}(t,x,d_2(t))f(t,x,d_1(t))\phi_{2t}(t,x),
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
P'_t &= \phi_{1tt} - L_{xt}(\phi_1(t,x)) - (\beta_{1t}(t,x) + d''_1(t))u_z(t,x,d_1(t)) - \\
&- \beta_1(t,x)(u_{zt}(t,x,d_1(t)) + u_z(t,x,d_1(t))d'_1(t)) - \\
&- \beta_{2t}(t,x)\phi_1^2(t,x) - 2\beta_2(t,x)\phi_1(t,x)\phi_{1t}(t,x) - \\
&- (u_{zt}(t,x,d_1(t)) + u_z(t,x,d_1(t))d'_1(t))d'_1(t),
\end{aligned}$$

$$\begin{aligned}
Q'_t &= \phi_{2tt} - L_{xt}(\phi_2(t,x)) - (\beta_{1t}(t,x) + d''_2(t))u_z(t,x,d_2(t)) - \\
&- \beta_1(t,x)(u_{zt}(t,x,d_2(t)) + u_z(t,x,d_2(t))d'_2(t)) - \\
&- \beta_{2t}(t,x)\phi_2^2(t,x) - 2\beta_2(t,x)\phi_2(t,x)\phi_{2t}(t,x) - \\
&- (u_{zt}(t,x,d_2(t)) + u_z(t,x,d_2(t))d'_2(t))d'_2(t),
\end{aligned}$$

$$\begin{aligned}
L_{xt}(\phi_1(t,x)) = \sum_{i,j=1}^{n} &\Big((\alpha_{ij})'\frac{\partial^2\phi_1}{\partial x_i \partial x_j} + \alpha_{ij}\frac{\partial\phi_1}{\partial x_i \partial x_j \partial t}\Big) + \\
&+ \sum_{i=1}^{n}\Big((\alpha_i)'\frac{\partial\phi_1}{\partial x_i} + \alpha_i\frac{\partial^2\phi_1}{\partial x_i \partial t}\Big),
\end{aligned}$$

$$L_{xt}(\phi_2(t,x)) = \sum_{i,j=1}^{n}\Big((\alpha_{ij})'\frac{\partial^2\phi_2}{\partial x_i\partial x_j} + \alpha_{ij}\frac{\partial\phi_2}{\partial x_i\partial x_j\partial t}\Big) + \\ + \sum_{i=1}^{n}\Big((\alpha_i)'\frac{\partial\phi_2}{\partial x_i} + \alpha_i\frac{\partial^2\phi_2}{\partial x_i\partial t}\Big).$$

By virtue of (6), (35)

$$|M_1(t,x)| \leqslant K_1, \quad |M_2(t,x)| \leqslant K_2, \tag{37}$$

here K_1, K_2 - are constants depending on δ_1, δ_2, C.

We integrate expressions (36) with respect to t in the range from 0 to t, we obtain

$$N_1(t,x)u(t,x,z) = N_1(0,x)u(0,x,z) + \int_0^t M_1(\eta,x)d\eta,$$

$$N_2(t,x) = N_2(0,x) + \int_0^t M_2(\eta,x)d\eta.$$

By virtue of (5), (37) $N_1(t,x)u(t,x,z) \geqslant \delta_1 - K_1 t$, $N_2(t,x) \geqslant \delta_2 - K_2 t$

$$N_1(t,x)u(t,x,z) \geqslant \frac{\delta_1}{2}, \quad N_2(t,x) \geqslant \frac{\delta_2}{2}, \quad t\in[0,t^*]. \tag{38}$$

By the definition of the cutoff function (15) and (38), we obtain $S_{\delta_1}(N_1(t,x)u(t,x,z)) = N_1(t,x)u(t,x,z)$, and $S_{\delta_2}(N_2(t,x)) = N_2(t,x)$ with $t\in[0,t^*]$, $t^* = \min\Big(t_*, \frac{\delta_1}{2K_1}, \frac{\delta_2}{2K_2}\Big)$, t_* see in (31).

Thus, in the equation (16), the cutoffs are removed. The function $u(t,x,z)$ satisfies the equation (12).

The coefficients $a(t,x)$ and $b(t,x)$ can be written in the form (11).

Thus, we have proved the existence of a solution $u(t,x,z)$ of the direct problem (12), (13) in the class $C^{1,2,4}_{t,x,z}(G_{[0,t^*]})$.

Let us prove that the triple of functions $u(t,x,z), a(t,x), b(t,x)$ is the solution of the inverse problem (1) – (3), where $a(t,x)$ and $b(t,x)$ are defined in (11). Since $u(t,x,z)$ is the solution of the direct problem (12), (13), substituting $u(t,x,z)$, $a(t,x)$, $b(t,x)$ in (1), we obtain the correct identity.

According to (6), (35) from (11), (12), we obtain that the triple of functions $u(t,x,z)$, $a(t,x)$, $b(t,x)$ belongs to the class (7) and satisfies the inequalities (8), (9).

Using conditions (4) and equation (1), we can prove that the overdetermination conditions (3) are satisfied.

The existence in the class $Z(t^*)$ of the solution $u(t,x,z)$, $a(t,x)$, $b(t,x)$ of problem (1) – (3) satisfying relations (1) – (3) is proved.

The uniqueness of the solution is proved in the standard way. It is assumed that there are two solutions to the original problem (1) – (3) that satisfy conditions (8), (9), and it is proved that the difference of these solutions is zero.

Thus the theorem is proved.

5 Example

In $G_{[0,1]} = \{(t,x,z)\big|\, 0 \leqslant t \leqslant 1,\, x \in E_1,\, z \in E_1\}$ we have the Cashy problem

$$\frac{\partial u}{\partial t} = (t^2+1)u_{xx}+(t+1)u_x+a(t,x)uu_{zz}+\sin(t+x)u_z+(3+\cos(x+t))u^2+ \\ + b(t,x)(-10\sin(t+z)+\sin(t+x+z)), \quad (39)$$

$$u(0,x,z) = u_0(x,z) = \cos x + \cos z + 4, \quad (x,z) \in E_2, \qquad (40)$$

$$f(t,x,z) = (-10\sin(t+z)+\sin(t+x+z)), \alpha_{11}(t) = t^2+1, \alpha_1(t) = t+1, \\ u_0(x,z) = \cos x + \cos z + 4, \beta_1(t,x) = \sin(t+x), \beta_2(t,x) = 3+\cos(t+x).$$

We assume, that overdeterminition conditions are hold on two different hyperspace $z = d_1(t) = -t^2$ and $z = d_2(t) = -t^2+1$:

$$\begin{aligned} u(t;x;-t^2) = \phi_1(t,x) = \cos(t+x)+\cos(t-t^2)+4, \\ u(t;x;-t^2+1) = \phi_2(t,x) = \cos(t+x)+\cos(t-t^2+1)+4, \end{aligned} \qquad (41)$$

where $(t,x) \in \Pi_{[0,1]}$, $\Pi_{[0,1]} = \{(t,x)|\, 0 \leqslant t \leqslant 1,\, x \in E_1\}$ and $\phi_1(t,x)$, $\phi_2(t,x)$ — satysfied of

$$\phi_1(0,x) = u_0(x,0) = \cos x + 5, \phi_2(0,x) = u_0(x,1) = \cos x + \cos 1 + 4,$$

where $x \in E_1$.

The sought coefficients are

$$a(t,x) = \frac{N_1(t,x)}{N_2(t,x)}, \quad b(t,x) = \frac{N_3(t,x)}{N_2(t,x)},$$

here

$$\begin{aligned} N_1(t,x) = (&-\sin(t+x)-\sin(t-t^2)(1-2t)+(t^2+1)\cos(t+x)+(t+1)\cdot \\ &\cdot \sin(t+x)+(\sin(t+x)-2t)\sin(t-t^2)-(3+\cos(t+x))\cdot \\ &\cdot (\cos(t+x)+\cos(t-t^2)+4)^2)(-10\sin(t-t^2+1)+ \\ &+\sin(t+x-t^2+1))-(-\sin(t+x)-\sin(t-t^2+1)(1-2t)+ \\ &+(t^2+1)\cos(t+x)+(t+1)\sin(t+x)+(\sin(t+x)-2t)\cdot \\ &\cdot \sin(t-t^2+1)-(3+\cos(t+x))(\cos(t+x)+\cos(t-t^2+1)+ \\ &+4)^2)(-10\sin(t-t^2)+\sin(t-t^2+x)), \end{aligned}$$

$$\begin{aligned} N_2(t,x) = -(&\cos(t+x)+\cos(t-t^2)+4)\cos(t-t^2)(-10\sin(t-t^2+1)+ \\ &+\sin(t+x-t^2+1))+(\cos(t+x)+\cos(t-t^2+1)+4)\cdot \\ &\cdot \cos(t-t^2+1)(-10\sin(t-t^2)+\sin(t+x-t^2)), \end{aligned}$$

$$\begin{aligned}N_3(t,x) = {}&(-\cos(t+x) - \cos(t-t^2) - 4)\cos(t-t^2)(-\sin(t+x)-\\&-\sin(t-t^2+1)(1-2t) + (t^2+1)\cos(t+x) + (t+1)\sin(t+x)+\\&+(\sin(t+x)-2t)\sin(t-t^2+1) - (3+\cos(t+x))(\cos(t+x)+\\&+\cos(t-t^2+1)+4)^2) + (\cos(t+x)+\cos(t-t^2+1)+4)\cdot\\&\cdot\cos(t-t^2+1)(-\sin(t+x)-\sin(t-t^2)(1-2t)+\\&+(t^2+1)\cos(t+x) + (t+1)\sin(t+x) + (\sin(t+x)-2t)\cdot\\&\cdot\sin(t-t^2) - (3+\cos(x+t))(\cos(t+x)+\cos(t-t^2)+4)^2).\end{aligned}$$

The input data are fairly smooth, have all continuous derivatives from (6).

Functions $N_1(0,x)u(0,x,z)$, $N_2(0,x)$ from (5) in $\Pi_{[0,1]}$ have the following form

$$\begin{aligned}N_1(0,x)u(0,x,z) = {}&((\cos(x) - (3+\cos(x))(\cos(x)+5)^2)(-10\sin(1)+\\&+\sin(x+1)) - (-\sin(1)+\cos(x)+\sin(x)\sin(1)-\\&-(3+\cos(x))(\cos(x)+\cos(1)+4)^2)\sin(x))\cdot\\&\cdot(\cos x + \cos z + 4) \geqslant \delta_1 \approx 600,\\N_2(0,x) = {}&-(\cos(x)+5)(-10\sin(1)+\sin(x+1)) + (\cos(x)+\\&+\cos(1)+4)\cos(1)\sin(x) \geqslant \delta_2 \approx 37.\end{aligned}$$

Therefore, the conditions (6), (5) are fulfilled.

The solution of (39) – (41) is a triple of functions $u(t,x,z)$, $a(t,x)$, $b(t,x)$, which have the form

$$a(t,x) = \frac{N_1(t,x)}{N_2(t,x)}, \quad b(t,x) = \frac{N_3(t,x)}{N_2(t,x)},$$
$$u(t,x,z) = \cos(t+x) + \cos(t+z) + 4.$$

6 Conclusion

In the presented article, a theorem of existence and uniqueness of the classical solution in the class of smooth bounded functions of the problem of determining the source function and the coefficient in the product of an unknown function and its second derivative with respect to a spatial variable in a multidimensional quasilinear parabolic equation is proved.

An example of input data satisfying the conditions of the proved theorem of existence and uniqueness of the classical solution of the posed inverse problem is given.

The obtained results are new, have theoretical value and can be used in the construction of a general theory of inverse problems.

References

[1] A. Fridman, *Partial differential equations of parabolic type*, Prentice-Hall, Englewood Cliffs, 1964. Zbl 0144.34903

[2] N.L. Gol'dman, *On a class of inverse problems with Cauchy data for a quasilinear parabolic equation*, Num. Meth. Appl., **5**:1 (2004), 70–82.
[3] N.L. Gol'dman, *Determination of the right-hand side in a quasilinear parabolic equation with a terminal observation*, Differ. Equ., **41**:3 (2005), 384–392. Zbl 1084.35121
[4] N.L. Gol'dman, *Inverse problems with final overdetermination for parabolic equations with unknown coefficients multiplying the highest derivative*, Dokl. Math., **83**:3 (2011), 316–320. Zbl 1232.35194
[5] A.D. Iskenderov, *Über ein inverses Problem für quasilineare parabolische Gleichungen*, Differ. Uravn., **10**:5 (1974), 890–898. Zbl 0285.35040
[6] A.D. Iskenderov, *Multidimensional inverse problems for linear and quasi-linear parabolic equations*, Sov. Math., Dokl., **16** (1975), 1564–1568. Zbl 0331.35052
[7] S.I. Kabanikhin, I.V. Koptyug, K.T. Iskakov, R.Z. Sagdeev, *Inverse problem for a quasi-linear equation of diffusion*, J. Inverse Ill-Posed Probl., **6**:4 (1998), 335–351. Zbl 0915.35115
[8] A. Lorenzi, *An inverse problem for a quasilinear parabolic equation*, Ann. Mat. Pura Appl., IV. Ser., **142** (1985), 145–169. Zbl 0604.35075
[9] V.M. Volkov, *An inverse problem for a quasilinear parabolic equation*, Differ. Uravn., **19**:12 (1983), 2166–2169. Zbl 0565.35106
[10] Yu.Ya. Belov, S.A. Kantor, *The weak approximation method*, Krasnoyarsk State University, Krasnoyarsk, 1999.
[11] Yu.Ya. Belov, I.V. Frolenkov, *Coefficient identification problems for semilinear parabolic equations*, Dokl. Math., **72**:2 (2005), 737–739. Zbl 1130.35375
[12] A.V. Datsenko, S.V. Polyntseva, *On the problem of identification of two lower coefficients and the coefficient by the derivative with respect to time in the parabolic equation*, J. Sib. Fed. Univ., Math. Phys., **5**:1 (2012), 63–74. Zbl 1521.35201
[13] A.M. Il'in, A.S. Kalashnikov, O.A. Oleĭnik, *Second order linear equations of parabolic type*, Russ. Math. Surv., **17**:3 (1962), 1–146. Zbl 0108.28401
[14] B.L. Rozhdestvenskij, N.N. Yanenko, *Systems of quasilinear equations and their applications to gas dynamics*, Nauka, Moscow, 1978. Zbl 0544.35001
[15] L.V. Kantorovich, G.P. Akilov, *Functional analisys*, Pergamon Press, Oxford etc., 1982. Zbl 0484.46003

Kira Ivanovna Ischenko
Siberian Federal University,
79 Svobodnyy pr.,
660041, Krasnoyarsk, Russia
Email address: ki.spirina@gmail.com

Svetlana Vladimirovna Polyntseva
Krasnoyarsk, Russia
Email address: svpolyntseva@gmail.com