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MULTILINEAR MULTIMODAL LOGIC WITH SEMI-RELIABLE INFORMATION OPERATOR

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Abstract: In this article we continue series of our works for the study of various logical systems and their possible interpretation in the field of computer science [1, 2]. Our goal is to expand the results that were presented in the article [2]. We also wish to show another look at the models with multivaluations that were presented in [3]. The main purpose of our work is to prove that the proposed logical system is decidable. At the end of the paper we will also provide an overview of the open problems and possible studies of such system.

Keywords: modal logic, knowledge, reliable information, temporal logic.

1 Introduction

It is known that modal logic's, namely temporal logic and cognitive logic, has found wide application in the field of computer science. This may be illustrated work of Amir Pnueli [4] and the monograph by Feigin, Halpern, Moshes and Vardi [5] and [6]. In the field of formal verification, many other

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logical systems generated from $\mathcal{LTL}$ have appeared. For example, $\mathcal{ITL}$ (interval temporal logic) created by Ben Moszkowski [7].

The theory of epistemic logic for the analysis of complex computing devices is currently actively developing. In our opinion, in this area it is worth noting the work of our foreign colleagues who studied the logic of multi-agent systems Byzantine [8, 9].

Previously researchers have also considered questions of making rational decisions under uncertainty that are not related to probabilistic logic. For this purpose we can consider among other logic, the majority logic [10] and its predecessor, graded modal logic [11]. At the moment, we are also aware of works in which majority is understood in a non-standard sense as a modular quantifier [12, Section 4].

In our recent work, we have studied some logical systems that can be useful for analyzing implicit information. This work is a continuation of a series of works on this topic [1, 2, 3].

Now we want to propose a logical system in which it is possible to model the interaction between agents. A single operator who cannot precisely view the history of agents' actions. Also we can interact at some marked points in time. While the time states that are not available to the operator. Can have different powers (cycles). Starting with models that have an infinite base, we will show how to move on to models of finite size, and then to models of exponential size. Our reasoning will be accompanied by examples and pictures for a clear idea of what we are talking about. Basically, we will touch upon the semantic approach in the sense of Kripke models. Definitions which will be given also are needed for a more complete understanding (we advise you to read the following monographs [13, 14, 15]).

2 Logic of $\mathbf{MLinML}_N$

2.1. Syntax. The alphabet of our logic uses propositional letters. Let $Prop = \{p, p_1, p_2, \dots\}$ be at most a countable set of variables. The language of logic is described as follows:

$$\mathcal{L}^{\mathbf{MLinML}_N} := \langle \vee^2, \wedge^2, \neg^1, K_1^1, \dots, K_N^1, \mathcal{N}_1^1, \dots, \mathcal{N}_N^1, \mathcal{N}_{Ac}^1, K_{Ac}^1 \rangle$$

Here are the rules for constructing formulas:

$$\varphi ::= p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \neg\varphi \mid K_i\varphi \mid K_{Ac}\varphi \mid \mathcal{N}_i\varphi \mid \mathcal{N}_{Ac}\varphi$$

where $i \in [1, N]$. Let $\hat{K}_{sy}\varphi := \neg K_{sy}\neg\varphi$, where $sy \in \{1, \dots, N, Ac\}$

The set of all $\mathcal{L}^{\mathbf{MLinML}_N}$ formulas is denoted by $Form$. For a formula φ, $Var(\varphi)$ denotes the set of all variables in φ.

2.2. Semantics. Let us describe the semantics of logic. First, we introduce the concept of a scale $\mathbf{MLinML}_N$

$$\mathcal{F}_N := \langle W_N, \prec_1, \prec_2, \dots, \prec_N, Next_1, \dots, Next_N, Next_{Ac}, \prec_{Ac} \rangle$$

where $W_N := \bigcup_{k\in\omega}\bigcup_{i=1}^{N}\{a_k, a_{k_1^i}, a_{k_2^i}, \dots, a_{k+1}\}$. Let's adopt the following abbreviation:

$[a_k, a_{k+1}]_i = \{a_k, a_{k_1^i}, a_{k_2^i}, \dots, a_{k+1}\}$; $(a_k, a_{k+1}]_i = \{a_{k_1^i}, a_{k_2^i}, \dots, a_{k+1}\}$

Moreover, for W_N it is looks as follows:

1. $(\forall i, j \in [1, N])\,((i \neq j) \to [a_k, a_{k+1}]_i \bigcap [a_k, a_{k+1}]_j = \{a_k, a_{k+1}\})$
2. $(\forall k \in \omega)(\bigcap_{i=1}^{N}[a_k, a_{k+1}]_i = \{a_k, a_{k+1}\})$
3. $(\forall s, k \in \omega)(\forall i, j \in [1, N])(|s - k| \geq 2 \to [a_k, a_{k+1}]_i \bigcap [a_s, a_{s+1}]_j = \emptyset)$
4. $(\forall i, j \in [1, N])(\forall k \in \omega)\ ([a_k, a_{k+1}]_i \bigcap [a_{k+1}, a_{k+2}]_j = \{a_{k+1}\})$

Let $Ac = \{a_0, a_1, \dots\}$ be a set of shared access points. Now define binary relations.

- $\prec_{Ac}$ is a linear order on Ac such that $a_0 \prec_{Ac} a_1 \prec_{Ac} \dots$
- $Next_{Ac} := \{\langle a_i, a_{i+1}\rangle | i \in \omega\}$

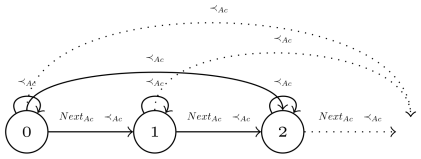

Fig. 1. Relations $\prec_{Ac}$ and $Next_{Ac}$

- $Next_i := \{\langle a_k, a_{k_0^i}\rangle | k \in \omega\} \bigcup \{\langle a_{k_z^i}, a_{k_{z+1}^i}\rangle | k, z \in \omega\}$

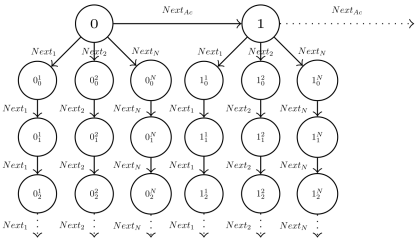

Fig. 2. Relations $Next_i$

- $\prec_i$ is a linear order on $\bigcup_{k\in\omega}\{a_k, a_{k_1^i}, \dots, a_{k+1}\}$ such that $a_0 \prec_i a_{0_0^i} \prec_i a_{0_1^i} \prec_i \dots \prec_i a_1 \prec_i \dots$

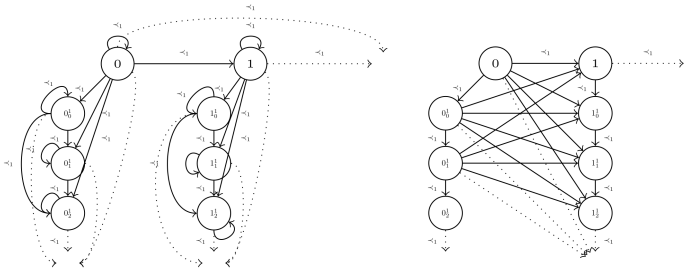

Fig. 3. Relation $\prec_1$

Now we move on to the definition our model $\mathcal{M}_N = \langle \mathcal{F}_N, V \rangle$, where $V : Prop \mapsto 2^{W_N}$. Let us introduce the rules for computing the truth (satisfiability) at the states of the model

- $(\mathcal{M}_N, x) \models_V p$ iff $x \in V(p)$
- $(\mathcal{M}_N, x) \models_V \neg\varphi$ iff $(\mathcal{M}_N, x) \not\models_V \varphi$
- $(\mathcal{M}_N, x) \models_V \varphi \wedge \psi$ iff $(\mathcal{M}_N, x) \models_V \varphi$ AND $(\mathcal{M}_N, x) \models_V \psi$
- $(\mathcal{M}_N, x) \models_V \varphi \vee \psi$ iff $(\mathcal{M}_N, x) \models_V \varphi$ OR $(\mathcal{M}_N, x) \models_V \psi$
- $(\mathcal{M}_N, x) \models_V K_i\varphi$ iff $(\forall y \in W_N)(x \prec_i y \to (\mathcal{M}_N, y) \models_V \varphi)$
- $(\mathcal{M}_N, x) \models_V \mathcal{N}_i\varphi$ iff $(\exists y \in W_N)(xNext_i y \wedge (\mathcal{M}_N, y) \models_V \varphi)$
- $(\mathcal{M}_N, x) \models_V \mathcal{N}_{Ac}\varphi$ iff $(\exists y \in W_N)(xNext_{Ac} y \wedge (\mathcal{M}_N, y) \models_V \varphi)$
- $(\mathcal{M}_N, x) \models_V K_{Ac}\varphi$ iff $(\forall y \in W_N)(x \prec_{Ac} y \to (\mathcal{M}_N, y) \models_V \varphi)$

Definition 1. *Let $\mathcal{K}_N$ be the class of all models of $\mathcal{M}_N$ defined above, then we define the logic $\boldsymbol{MLinML}_N$ as follows $\mathcal{L}_{\boldsymbol{MLinML}_N} := \{\varphi \in Form \mid \forall \mathcal{M} \in \mathcal{K}_N \quad \mathcal{M} \models_V \varphi\}$.*

Theorem 1. $\mathcal{L}_{\boldsymbol{MLinML}_1} \subseteq \mathcal{L}_{\boldsymbol{MLinML}_2} \subseteq \cdots \subseteq \mathcal{L}_{\boldsymbol{MLinML}_K}$, *where* $K \in \omega$

Proof this theorem complied below.

3 Examples. Operator of semi-reliability of information (SR)

As it was written in the introduction, such a system describes situations well enough when the moderator of the system has a low level of access to interact with agents.

Now we want to introduce an operator of semi-reliable information. The definition of such operator as the reliability of agents and the system as a whole can only be performed by a moderator (and he can receive information only from the recent past). This is exactly what the operator describes below:

$$(\mathcal{M}_N, x) \models_V SR_i(\varphi) \text{ iff}$$
$$(\exists y \in W_N)(yNext_{Ac}x) \wedge (\exists z \in [y; x)_i \forall k \in [z; x)_i : (\mathcal{M}_N, k) \models_V \varphi)$$

The second thing that also makes sense is that it is possible to consider a multi-valued model: $\mathcal{M}^{mv}_{N,k} = \langle \mathcal{F}_N, V_1, ..., V_k \rangle$, where $V_1, \ldots, V_k$ are memory areas, if earlier we considered models where $V_1, \ldots, V_k$(valuations) are the opinions/points of view of agents. We assume that these are memory cells that the agents wrote down, and then we can introduce the following rules for calculating satisfiability:

$$(\mathcal{M}_{N,k}, x) \models p^i \text{ iff } x \in V_i(p), \text{where } i \in [1, k]$$

Then we can make, for example, the following formulas:

$(\mathcal{M}_{2,2}, x) \models K_1 p^1 \wedge K_1 \neg p^2$ - Agent No. 1 knows that the information in memory cell No. 1 is true, and in cell No. 2 it is absurd.

$$\mathcal{L}_{\mathbf{MLinML}_{N,k}} = \{\varphi \in Form \mid \forall \mathcal{M}_{N,k} [\mathcal{M}_{N,k} \models \varphi]\}$$

In this article, we will not further explore the multi-valued system, but nevertheless we believe that this is an interesting area for further study. Also note that in this work we have in some sense expanded the concept of interval. Therefore in the case of initially finite chains inside bundles, we could additionally introduce the operator D from [1].

(1) $(\mathcal{M}_3, x) \models_V K_{Ac}SR_1\varphi \wedge K_{Ac}\neg\varphi$

So agent 1 contradicts the moderator. Or it can be interpreted as follows: the information coming from agent 1 is unreliable.

(2) $(\mathcal{M}_3, x) \models_V K_{Ac}SR_1\varphi \wedge K_{Ac}SR_2\varphi \wedge K_{Ac}\varphi$

The information φ in this system is almost reliable.

4 Finite Model Property

Let Λ be a normal modal logic, M a set of finite-base models such that $\Lambda = \Lambda_M$, and f a function mapping natural numbers to natural numbers. Λ has the finite $f(n)$-model property with respect to M if every Λ-consistent formula φ is satisfiable in a model in M containing at most $f(|\varphi|)$ states.

Λ has the **strong finite model property** with respect to M if there exists a computable function f such that Λ has the $f(n)$-model property with respect to M.

By the length of a formula $|\varphi|$ we mean the total number of occurrences of logical connectives and propositional variables, including those with repetitions, in it.

Theorem 2. *If Λ is a normal modal logic that has the strong finite model property with respect to a recursive set of models M, then Λ is decidable.*[13]

In order to establish the fact that logic has the finite model property, we prove several auxiliary lemmas.

Definition 2. *Let $NFA(\varphi) : \varphi \mapsto \omega$ be a function that computes the number of distinct indices under the operators $K_{sy}, \mathcal{N}_{sy}$ in the formula φ, where $sy \in \{1, \ldots, N, Ac\}$.*

Algorithm for calculating $NFA(\varphi)$:

1. Create a list $List$ of size N (counting from 1) of type Boolean (logical type that takes the values True or False). Initially, all N elements of the list take the value $False$.

2. Next, we iterate over the elements of the formula. If the i-th element of the formula is K_{sy} or $\mathcal{N}_{sy}$, then depending on the value of sy, we change the value in $List[sy] := True$, if $sy == Ac$ then we do nothing.

3. Count the number of elements in $List$ that are different from $False$, this number will be the value of the function $NFA(\varphi)$.

Let $\mathcal{A} = \{1, ..., N, Ac\}$ and $\mathcal{FA}$ be all such values of indices of elements of $List$ from the algorithm above that were equal to $False$.

Definition 3. *For the model $\mathcal{M}_N$, let's call by the model reduced by φ the model*

$$\langle W_{NFA(\varphi)}, \prec_1, ..., \prec_{NFA(\varphi)}, Next_1, ..., Next_{NFA(\varphi)}, Next_{Ac}, \prec_{Ac}, V_{NFA(\varphi)} \rangle$$

where

- $W_{NFA(\varphi)} := W \setminus (\cup_{k\in\omega}[a_k; a_{k+1}]_{i\in\mathcal{FA}}) \cup Ac$
- $V_{NFA(\varphi)}(p) := V(p) \cap 2^{W_{NFA(\varphi)}}\ \forall p \in Var(\varphi)$
- $R_i^{NFA} = R_i \cap W_{NFA(\varphi)} \times W_{NFA(\varphi)}$, *where R - binary relation*

Lemma 1. $(\forall\varphi \in Form)\ (\mathcal{M}_N, x) \models_V \varphi \Rightarrow (\mathcal{M}_{NFA(\varphi)}, x) \models_V \varphi$

Proof. We will prove it by induction on the length of formula φ.

For propositional variables, it is true by assumption and definitions. Inductive step is obvious for logical operations of classical logic.

Let $(\mathcal{M}_N, x) \models_V K_i\varphi$. Then for all $\forall y \in W_N\ x \prec_i y \to (\mathcal{M}_N, y) \models_V \varphi$. We constructed the model in such a way that if the sub-formula in the formula was the i-th index, then all relations and all states are preserved. Then $(\mathcal{M}_{NFA(\varphi)}, y) \models_V \varphi$. Therefore $(\mathcal{M}_{NFA(\varphi)}, x) \models_V K_i\varphi$. This argument is true for all other connectives. □

For example, to carry out such a reduction in [2] if would be difficult from a technical point of view due to the fact that all the designations affected the value of the formula $AP(\varphi)$

Lemma 2. $(\forall\varphi \in Form)\ (\mathcal{M}_{NFA(\varphi)}, x) \models_V \varphi \Rightarrow (\mathcal{M}_N, x) \models_V \varphi$, *for all N such that $N \geq NFA(\varphi)$*

Proof. To do this, we need to build our model step by step. At each stage, we add new chains numbered $NFA(\varphi) + 1$ and set an arbitrary value of propositional variables on the new states. □

We introduce a couple more definitions that will be used to prove the theorem below.

Definition 4. *For each $C \subseteq W$, C is a cluster if:*

- $\forall a, b \in C\ (aRb) \wedge (bRa)$
- $\forall a \in C\ \forall c \in W\ (aRc) \wedge (cRa) \to (c \in C)$

Definition 5. $Theory^{\varphi}(x) = \{\alpha \in Sub(\varphi) \mid x \models \alpha\ \}$ *and* $Opport_i^{\varphi}(x) = \{Theory^{\varphi}(y) | x \prec_i y\}$ *respectively.*

Lemma 3. $(\forall i \in [1, NFA(\varphi)])(\exists x \in [a_1, \infty]_i)(\forall y \in W_{NFA(\varphi)})(x \prec_i y \to Opport_i^{\varphi}(x) = Opport_i^{\varphi}(y))$

Proof. This lemma follows from the observation that the formula φ has finite length and the number of subformulas of the formula in subsequent states cannot increase. □

Lemma 4. $(\exists x \in Ac)(\forall i \in [1, NFA(\varphi)])(\forall y \in W_{NFA(\varphi)})(x \prec_i y \to Opport_i^{\varphi}(x) = Opport_i^{\varphi}(y))$

Proof. Follows immediately from Lemma 3 □

Definition 6. *The interval* $[a, b]$ *has the stabilization property if* $(\forall x)(\exists c \in [a, b])(b \prec x) \to (Theory^{\varphi}(x) = Theory^{\varphi}(c))$.

Definition 7. *A stabilization cluster is a cluster generated by a stabilization interval. We will denote such cluster* C_{stable}.

Consider a model of the form

$$\mathcal{M}^{fin_{Ac}}_{NFA(\varphi)} = \left\langle \bigcup_{i=1}^{NFA(\varphi)} [a_0, stable_2]_i \bigcup C_{stable_{Ac}}, \{\prec_i^{NFA}, Next_i^{NFA}\}_{i \in \mathcal{NFA}}, V_{NFA(\varphi)} \right\rangle$$

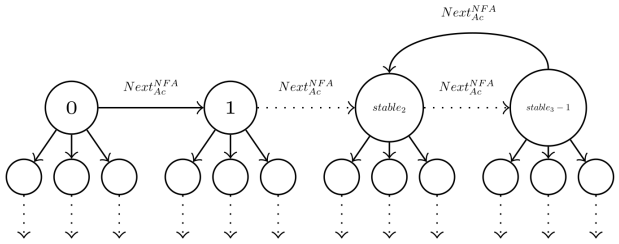

Fig. 4. Example $\mathcal{M}^{fin_{Ac}}_{NFA(\varphi)}$

Below we will give an algorithm by which we can move to such a model from the model $\mathcal{M}_{NFA(\varphi)}$ and vice versa.

Lemma 5. *If* $\varphi \in Form$ $(\mathcal{M}_{NFA(\varphi)}, x) \models_V \varphi$ *then* $(\mathcal{M}^{fin_{Ac}}_{NFA(\varphi)}, x) \models_V \varphi$

Proof. Apply Lemma 4, call the resulting state $stable_1 \in Ac$.

1. We find $stable_2$: $stable_1 \prec_{Ac} stable_2$ and $[stable_1, stable_2]_i$ is stabilization interval for all $i \in [1, NFA(\varphi)]$.

2. The we find $stable_3$: $stable_2 \prec_{Ac} stable_3$ and $[stable_2, stable_3]_i$ is stabilization interval for all $i \in [1, NFA(\varphi)]$. And

$$Theory^{\varphi}(stable_2) = Theory^{\varphi}(stable_3)$$

and

$$(\forall i \in [1, NFA(\varphi)])(Theory^{\varphi}(Next_i(stable_2)) = Theory^{\varphi}(Next_i(stable_3)))$$

and

$$Theory^{\varphi}(Next_{Ac}(stable_2)) = Theory^{\varphi}(Next_{Ac}(stable_3)).$$

3. We delete all states that are further away than $stable_3$.

4. $Next_{Ac}(stable_3) = Next_{Ac}(stable_2)$. That is, we glue the points $stable_2$ and $stable_3$. Thus, we have obtained a cluster of stabilization with respect to Ac.

The proof is by induction on the length of the formula. It is easy to understand that we have not affected the truth values inside the circuits, but we need to check K_{Ac} and $\mathcal{N}_{Ac}$ and SR_i.

Assume $(\mathcal{M}_{NFA(\varphi)}, x) \models_V SR_i(\varphi)$ then by definition
$(\exists y \in W_{NFA(\varphi)})(yNext_{Ac}x) \wedge (\exists z \in [y;x)_i \forall k \in [z;x)_i : (\mathcal{M}_{NFA(\varphi)}, k) \models_V \varphi)$ but to construct the model $\mathcal{M}^{fin_{Ac}}_{NFA(\varphi)}$ we chose the points in such a way that the stabilization property is fulfilled, which means that in this model there will exist the same state as in the first model such that the truth values on the chain are preserved. □

Lemma 6. *If* $\varphi \in Form$ *and* $(\mathcal{M}^{fin_{Ac}}_{NFA(\varphi)}, x) \models_V \varphi$ *then* $(\mathcal{M}_{NFA(\varphi)}, x) \models_V \varphi$.

Now we only have non-finite chains that are generated by states. In fact, the same method can be applied to them with the same conditions as was alone in [2], with one condition that we do not consider the entire state of the agent, but from one access states to the next.

Define the model $\mathcal{M}^{fin_{Ac\&ch}}_{NFA(\varphi)}$ as a model in which each chain starting from the Ac points consists of a part c of a straight line plus a circle, like the Ac itself, i.e. we also select three stabilization points and get an interval plus a stabilization cluster.

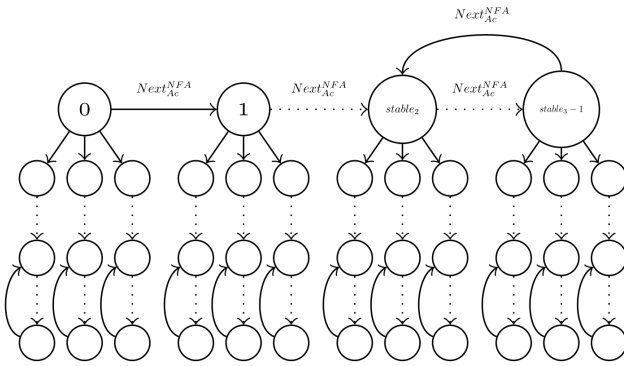

Fig. 5. Example $\mathcal{M}^{fin_{Ac\&ch}}_{NFA(\varphi)}$

Lemma 7. $\varphi \in Form$ $(\mathcal{M}^{fin_{Ac}}_{NFA(\varphi)}, x) \models_V \varphi$ *iff* $(\mathcal{M}^{fin_{Ac\&ch}}_{NFA(\varphi)}, x) \models_V \varphi$

Proof. The methodology for constructing such models was described for example in [2]. □

Theorem 3. $\mathcal{L}_{\boldsymbol{MLinML}_N}$ *has the strong finite model property and as a consequence is decidable.*

Proof. First, we must reduce the total number of points from Ac to a number efficiently calculated from the length of the formula.

Let's look at $C_{stable_{Ac}}$:

We generated this cluster with the states $\bigcup_{i=1}^{NFA(\varphi)}[stable_2, stable_3]_i$. We consider all states starting from $stable_2$ until we get back to $stable_2$. That is $x := stable_2$ and $y := Next_{Ac}(x)$.

IF $(\forall i \in [1, NFA(\varphi)])(\forall k \in C_{stable_{Ac}})(\exists c \in [x,y]_i)(Theory^{\varphi}(c) = Theory^{\varphi}(k))$ AND $Theory^{\varphi}(Next_{Ac}(x)) = Theory^{\varphi}(Next_{Ac}(y))$

THEN delete $(x, y)_i$ for all i and glue x and y.
$y := Next_{Ac}(y)$.

Then, when y has passed a full circle, we move the point x and repeat this procedure until x comes back to the starting point $stable_2$.

We do the same with the part that is not of the cluster. This procedure allows you to limit the number of points Ac by the following function: $2^{1+2^{|\varphi|}}$. Next, we can do the same procedure with all chains.

The overall estimate looks like this:
$2^{1+2^{|\varphi|}} \cdot (NFA(\varphi) \cdot 2^{1+2^{|\varphi|}}) = NFA(\varphi) \cdot 2^{2+2^{1+|\varphi|}} \leq |\varphi| \cdot 2^{2^{2+|\varphi|}}$ □

5 Some open problems

Hypothesis 1. *Logic $\mathcal{L}_{\boldsymbol{MLinML}_{N,k}}$ is decidable. Also the same problem with AP for valuations* [2].

To do this, you may need to extend previous techniques.

Hypothesis 2. *Using the multivalue technique*[3]*, it is possible to simulate the logic of $\mathcal{L}_{\boldsymbol{MLinML}_N}$ without introducing additional binary relations and possibly even on linear models.*

6 Conclusion

In this short article, we explored multilinear logic with an additional operator. Such models illustrate the interaction between N agents and a single moderator. We proved that the proposed logic $\mathcal{L}_{\boldsymbol{MLinML}_N}$ is decidable.

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